Simple model for Maxwell's-demon-type information engine

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We have investigated the recently proposed self-consistent theory of fluctuation-induced transport. In this framework the subsystem under study is coupled to two independent baths at different temperatures. In this nonequilibrium system one can extract energy at the expense of increased entropy. This is a simple model of Maxwell's-demon-type engine that extracts work out of a nonequilbrium bath by rectifying internal fluctuations. We point out an error in the earlier results. We have obtained an analytical expression for the fluctuation-induced transport current in a nonequilibrium state that is valid at any temperatures, and various cases of physical interest have been elucidated.

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From thermodynamics, it is well known that useful work cannot be extracted from equilibrium fluctuations [1]. In a thermal equilibrium state the principle of detailed balance ensures that no net particle current can flow in the presence of external potential of arbitrary shape. In contrast, in a nonequilibrium situation, where detailed balance is lost, net current flow is possible, i.e., one can extract energy at the expense of increased entropy. Several models have been proposed recently in this direction [2–6]. The motion of a heavily damped Brownian particle, in the presence of asymmetric static external periodic potential and under nonwhite or correlated fluctuations, is a simple example of a nonequilibrium system. In such a system induced current or directed motion appears, even though the average of the driving fluctuations vanishes. It turns out that the preferred direction of motion and the magnitude of induced current depend sensitively on the shape of the potential as well as on the statistics of the fluctuations. Moreover, to obtain induced current the strength of nonwhite noise must exceed a certain minimum value and the magnitude of induced current shows a maximum value at an intermediate value of the noise strength. These models [2–7] of engines to obtain coherent response (or rectification) from unbiased forcings come under the common denomination of "thermal ratchets" or "fluctuation-induced transport systems." [Thermal ratchets refers to a thermodynamic mechanism (machine) that aids motion of a particle in an asymmetric periodic (sawtooth type) potential in the presence of thermal fluctuations.] The idea of thermal ratchets has been utilized recently for molecular separation [8]. One of the major motivations of these studies comes from molecular biophysics, where ratchetlike mechanism is proposed to explain unidirectional movement of macromolecules or molecular motors. This is a physical example of preferred directional motion of Brownian particles (macromolecules) along periodic structures in the absence of obvious driving potentials, such as chemical potential gradients or thermal gradients.

In a physically well-motivated recent work [9], Millonas points out that all the proposed earlier models [2–6] are basically phenomenological in nature and no attempt has been made to formulate the problem from first principles. Millonas in his treatment constructs a Maxwell's-demon-type information engine [9] that extracts work from a nonequilibrium bath and allows a rigorous determination of kinetics consistent with the underlying laws of physics. He explicitly writes down a microscopic Hamiltonian including the subsystem and two thermal baths at different temperatures. An existing inequality of temperatures is exploited to do useful work. After eliminating bath variables one obtains the nonlinear Langevin equation for the subsystem variable Q: namely,

$$M\ddot{Q}+\Gamma(Q)\dot{Q}+U'(Q)=\xi_A(t)+\sqrt{f(Q)}\xi_B(t)\ , \tag{1}$$

where $\Gamma(Q)=\Gamma_A+\Gamma_B f(Q)$, $\xi_A(t)$ and $\xi_B(t)$ are two independent Gaussian white-noise fluctuating forces with statistics given by

$$\langle \xi_A(t)\rangle=0\ ,$$
$$\langle \xi_A(t)\xi_A(t')\rangle=2\Gamma_A T\delta(t-t') \tag{2a}$$

and

$$\langle \xi_B(t)\rangle=0\ ,$$
$$\langle \xi_B(t)\xi_B(t')\rangle=2\Gamma_B k\bar{T}\delta(t-t')\ , \tag{2b}$$

T and $\bar{T}$ are temperatures of the two baths A and B, respectively. Henceforth we set the Boltzmann constant k to unity. The bath B is characterized by a space-dependent friction coefficient $\Gamma_B f(Q)$. In Eq. (1) $U'(Q)$ is an external force [correspondingly $U(Q)$ is an external potential] and we have used the expression $f(Q)$ for $[V'(Q)]^2$ of Ref. [9]. In Ref. [9] two different cases are considered. In the first case nonequilibrium baths are not in quasithermal equilibrium with one another and fluctuation-induced transport is derived without making use of the Fokker-Planck equation. In the second case, the nonequilibrium baths are taken to be in quasithermal equilibrium with each other and the results are based on

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the associated Fokker-Planck equation in the high damping limit. We would like to emphasize that the result for this second case is not correct for all temperatures (especially at high temperatures). The final calculations in Ref. [9] for the second case are done in an overdamped limit by simply neglecting $\ddot{Q}$ term in Eq. (1). We note that such a procedure is incorrect [10] and the resulting Fokker-Planck equation [Eq. (8) of Ref. [9]], turns out to be inconsistent. For example, in an equilibrium situation ($T=\bar{T}$), when the potential $U(Q)$ is unbounded and positive, i.e., $U(Q)\to\infty$ as $Q\to\pm\infty$, the equilibrium distribution comes out to be

$$P_c(Q)=C\Gamma(Q)e^{-U(Q)/T}, \qquad (3)$$

where C is the normalization constant. This distribution function is incorrect because in equilibrium $P_e(Q)$ must have the form

$$P_e(Q)=Ne^{-U(Q)/T}, \qquad (4)$$

where N is the position-independent normalization constant. Also, if we set $\Gamma_A=0$ (i.e., the particle is coupled to a single bath B at temperature $\bar{T}$) again the equilibrium distribution of the form of Eq. (3) is obtained. The reason for this inconsistency can be traced back to the improper overdamped limit of the original Langevin equation [Eq. (1)]. For example, in the absence of thermal bath A, Sancho, San Miguel, and Duerr [11] have shown that the correct overdamped Langevin equation should be

$$\Gamma_B f(Q)\dot{Q}=-U'(Q)-\bar{T}\frac{[\sqrt{f(Q)}]'}{\Gamma_B\sqrt{f(Q)}}+\sqrt{f(Q)}\xi_B(t). \qquad (5)$$

The above equation leads to the correct equilibrium distribution as mentioned above. The error in the Millonas treatment [9] follows from the incorrect overdamped limit, where he ignores a term like the second term on the right-hand side of Eq. (5).

In this Brief Report, we construct the correct Fokker-Planck equation in the overdamped limit. We use this equation to study fluctuation-induced transport at any temperature low as well as high in a system where the functions $U(Q)$ and $f(Q)$ are periodic under translation $Q\to Q+\lambda$ [$U(Q+\lambda)=U(Q)$ and $f(Q+\lambda)=f(Q)$]. We obtain the correct expression for the mean velocity $\langle\dot{Q}\rangle$ and study several special cases of physical interest.

Following Ref. [11], after a straightforward algebra one can readily obtain the Fokker-Planck equation for the variable Q [i.e., the evolution equation for the probability density $P(Q,t)$], in the overdamped limit and is given by

$$\frac{\partial P}{\partial t}=\frac{\partial}{\partial Q}\frac{U'(Q)}{\Gamma(Q)}P+T\Gamma_A\frac{\partial}{\partial Q}\frac{1}{\Gamma(Q)}\frac{\partial}{\partial Q}\frac{1}{\Gamma(Q)}P+\bar{T}\Gamma_B\frac{\partial}{\partial Q}\frac{\sqrt{f(Q)}}{\Gamma(Q)}\frac{\partial}{\partial Q}\frac{\sqrt{f(Q)}}{\Gamma(Q)}P+\bar{T}\Gamma_B\frac{\partial}{\partial Q}\frac{[\sqrt{f(Q)}]'\sqrt{f(Q)}}{[\Gamma(Q)]^2}P. \qquad (6)$$

This is the correct Fokker-Planck equation in the overdamped limit in place of Eq. (8) of Ref. [9] and represents the Smoluchowski approximation to the original Eq. (1). When the potential $U(Q)$ is unbounded and positive, i.e., $U(Q)\to\infty$ as $Q\to\pm\infty$, the system evolves towards the stationary distribution $P_s(Q)$. This stationary distribution is characterized by no net current flow and is given by

$$P_s(Q)=Ne^{-\Psi(Q)}, \qquad (7)$$

where N is normalization constant and

$$\Psi(Q)=\int^Q\left\{\frac{U'(x)\Gamma(x)}{[T\Gamma_A+\bar{T}\Gamma_B f(x)]}+\frac{(\bar{T}-T)}{\Gamma(x)}\frac{\Gamma_A\Gamma_B f'(x)}{[T\Gamma_A+\bar{T}\Gamma_B f(x)]}\right\}dx. \qquad (8)$$

One can readily notice that in the equilibrium situation, i.e., when $T=\bar{T}$, $P_s(Q)$ reduces to the correct equilibrium distribution as given in Eq. (4).

To study the case of fluctuation-induced transport, we take a simple case where both $U(Q)$ and $f(Q)$ are periodic functions and are invariant under the same transformation $Q\to Q+\lambda$. Now, the basic problem reduces to finding the mean velocity $\langle\dot{Q}\rangle$ of the subsystem given the shape of $U(Q)$ and $f(Q)$. Following the procedure of Refs. [9,12,13] closely, one can get the exact expression for the averaged velocity [in place of Eq. (9) of Ref. [9]]

$$\langle\dot{Q}\rangle=\frac{1-\exp(-\delta)}{\int_0^\lambda dy\exp-[\Psi(y)]\int_y^{y+\lambda}\{[\Gamma(x)]^2/[T\Gamma_A+\bar{T}\Gamma_B f(x)]\}\exp[\Psi(x)]dx}, \qquad (9)$$

where

$$\delta=\Psi(x)-\Psi(x+\lambda), \qquad (10)$$

and Ψ is given by Eq. (8). It is easy to see from Eqs. (9) and (10) that in the equilibrium case, when the temperature difference between the baths is zero ($T=\bar{T}$), the current vanishes identically (since $\delta=0$). Also, one can easily verify that when the subsystem is coupled to a single bath, i.e., when either Γ_A or Γ_B is zero, no net current is possible. It should be noted that the bath B, which gives rise to space-dependent friction coefficient $\Gamma_B f(Q)$ plays a special role. This can be noticed from the fact that if $f(Q)$ is independent of Q the induced current is zero. In the extreme case of high friction limit ($\Gamma_A \rightarrow \infty$ or $\Gamma_B \rightarrow \infty$) no net current is possible because the particle cannot execute the Brownian motion.

We consider the case where amplitude modulations in $f(Q)$ and $f'(Q)$ are small compared to the amplitude modulation of the potential $U(Q)$, the second term in Eq. (8) can be neglected. In this particular limit the problem becomes equivalent to a particle moving in a spatially varying temperature field, namely, $T(Q)=[T\Gamma_A + \bar{T}\Gamma_B f(Q)]/\Gamma(Q)$. It is well known in earlier literature [12–14] that such a spatial modulation of temperature field can give induced currents. Moreover, the problem of evolution of a particle in a spatially varying temperature field has a fundamental consequence in relation to local versus global stability criterion in statistical mechanics [15,16]. In a thermal equilibrium state the Boltzmann factor gives a relative occupation probability of states at different local stable points without invoking the behavior of the potential profile between the states. However, in the presence of spatially varying temperature field, the relative stability (or occupation probability) between different states depends sensitively on the intervening potential and more importantly one can control the relative stability among two different states by modifying the kinetics in the sparsely occupied intervening states [14,16].

In conclusion, we have reinvestigated the recently proposed self-consistent microscopic theory of fluctuation-induced transport [9]. We have pointed out an error in the earlier theory for the particular case: when two nonequilibrium baths are in quasithermal equilibrium with each other. We have obtained an analytical expression for fluctuation-induced current in a nonequilibrium situation valid at low as well as high temperatures and have discussed various cases of physical interest. Finally, we would like to mention that our expression for the stationary distribution (or steady state) $P_s(Q)$ [Eq. (7)] is not a local function of $U(Q)$ and $f(Q)$. In such a situation as discussed above the relative stability between two different local states in $U(Q)$ depends sensitively on the intervening behavior of $U(Q)$ and $f(Q)$. This can lead to interesting physics. For example, for given external potential $U(Q)$, as one varies physical parameters (in the parameter space of T, $\bar{T}$, Γ_A, *and* Γ_B) we expect additional maxima and minima to appear in $P_s(Q)$. Thus we can modify the stability properties of the subsystem. The qualitative changes in the stationary state of the subsystem are reflected in the behavior of the extrema of the $P_s(Q)$. Moreover, the extrema in $P_s(Q)$ may have little or no relationship to the extrema in the original potential $U(Q)$. Each new structure in $P_s(Q)$ may correspond to an entirely new state of a subsystem. In the spirit of the well-known noise-induced phase transitions this is equivalent to having a hierarchy of phase transitions, in a nonequilibrium system [17].

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